

An R Package for Adaptive Assessment Utilizing Knowledge Space Theory and Formal Psychological Assessment

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Outline

- 1 Introduction
- 2 mycaas Package
 - Algorithm
 - Shiny
 - Application on RAISE
- 3 Final Remarks

Tests in Education and Clinical Psychology

- Time consuming
- Fatigue effect, social desirability, etc.

(Informal) Definition

A computerized adaptive assessment is an evaluation that adjusts the difficulty and nature of subsequent questions based on the test-taker's responses to previous ones.

Tests in Education and Clinical Psychology

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(Informal) Definition

A computerized adaptive assessment is an evaluation that adjusts the difficulty and nature of subsequent questions based on the test-taker's responses to previous ones.

Pros of Adaptive Assessment

- Increased Efficiency and Accuracy in Assessment:
 - Adaptive systems save time by focusing on the appropriate difficulty level.
- Personalized Learning or Therapy :
 - Feedback can be customized to individuals need.
- Immediate Feedback:
 - Results are available as soon as the assessment is finished.



Cons of Adaptive Assessment

- Dependent on the assumptions of the Model :
 - The validity of results depends on the correctness of the model used.
- Complexity of Implementation:
 - Requires sophisticated algorithms and data processing infrastructure.
- Difficulty in Tuning:
 - Fine-tuning the assessment to achieve accurate difficulty adjustments is challenging.

My computerized adaptive assessment R package

mycaas package

```
devtools::install_github("brancaccioandrea/mycaas")
```

- Based on the strong theoretical foundation
- User-friendly graphical interface made
- Performance analysis to evaluate accuracy and efficiency

Theoretical Framework

Knowledge space theory (KST; Doignon & Falmagne, 1985):
The objective is to precisely describe what the individual knows (their knowledge state) in a given domain of knowledge, rather than computing a numerical score.

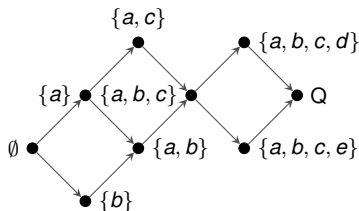
Formal Psychological Assessment (FPA; Spoto, Stefanutti & Vidotto, 2010): The objective from the clinical perspective is to give an in-depth evaluation of the construct investigated by the questionnaire.



Basic Definitions

Formal definitions of basic concepts encountered so far:

- **Domain** a either finite or infinite set Q of questions
- **State** the subset $K \subseteq Q$ of all questions that define the status of individual
- **Structure** a pair (Q, \mathcal{K}) , where \mathcal{K} is a collection of subset of Q , containing at least the empty set and Q



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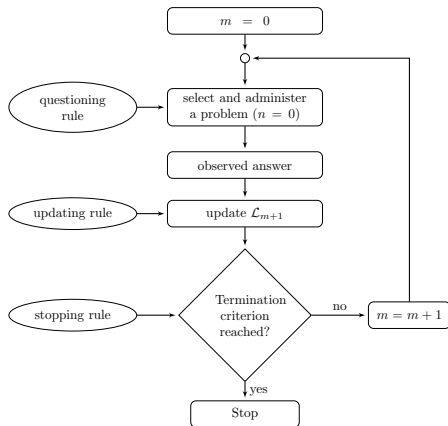
Flowchart of the Adaptive Assessment

Doignon & Falmagne, 1988; Donadello, et al., 2017

The goal of the assessment is to recover the **true state** of an individual asking the fewest possible questions

- Three rules guide the assessment:

- 1 Questioning rule
- 2 Updating rule
- 3 Termination rule

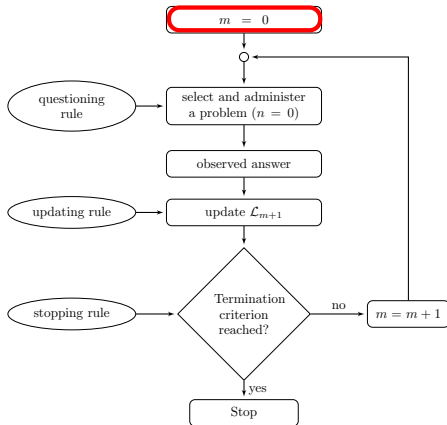


Probability distribution on the states

A probability distribution

$$\mathcal{L}_m : \mathcal{K} \rightarrow (0, 1)$$

Without prior knowledge is a uniform distribution



Questioning Rule

Half Split Rule

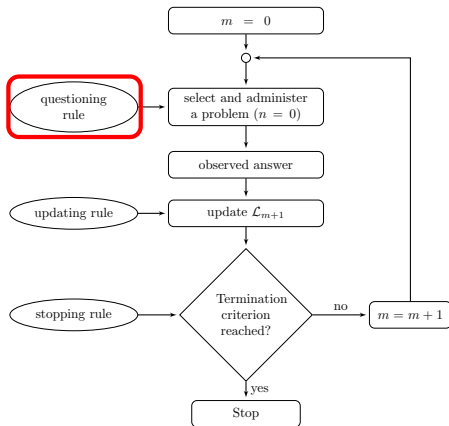
Select the “best” question to ask

Doignon & Falmagne, 1988

half_split: Let $\mathcal{L}_m(K)$ the probability of K at the step m , and the subset $\mathcal{K}_q \subset \mathcal{K}$ such that $q \in K$ for each $K \in \mathcal{K}_q$.

It selected problem $q \in Q$ that minimize

$$|\mathcal{L}_m(\mathcal{K}_q) - 1/2|$$



Updating Rule

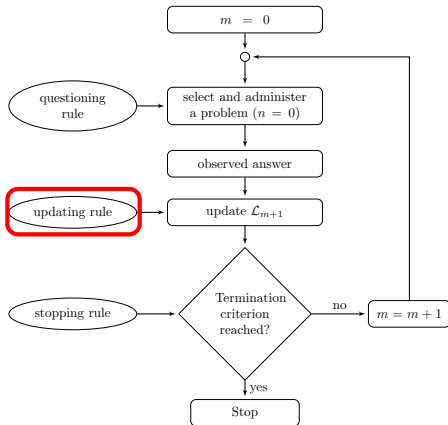
Bayesian Updating Rule

The probability $\mathcal{L}_m(K)$, for each $K \in \mathcal{K}$ is updated in function of the **the observed response** r_q collected for problem q as follows.

$$\mathcal{L}_{m+1}(K) = \frac{P(r_q|K)\mathcal{L}_m(K)}{\sum_{K' \in \mathcal{K}} P(r_q|K')\mathcal{L}_m(K')}$$

Parameters

$$P(r_q|K) = \begin{cases} \beta_q & \text{if } r_q = 0 \text{ \& } q \in K; \\ 1 - \eta_q & \text{if } r_q = 0 \text{ \& } q \notin K; \\ 1 - \beta_q & \text{if } r_q = 1 \text{ \& } q \in K; \\ \eta_q & \text{if } r_q = 1 \text{ \& } q \notin K. \end{cases}$$



Termination rule

Heller & Repitsch, 2012

Likelihood Maximization:
The assessment terminate at step m if

$$\max \mathcal{L}_m(K) > SC$$

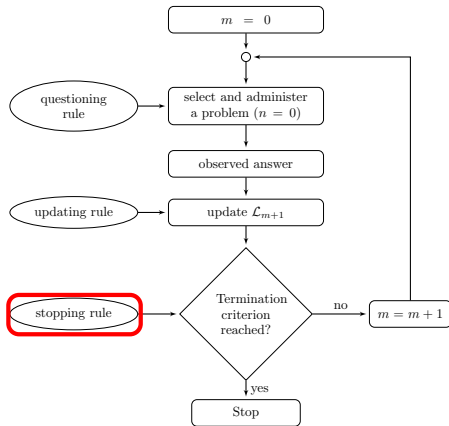
with $SC \in (0.5, 1]$.

Donadello, et al., 2017

Item Discrimination: The assessment terminate at step m if for each $q \in Q$

$$\mathcal{L}_m(K_q) > SC \text{ or } \mathcal{L}_m(K_q) < 1 - SC$$

with $SC \in [0.5, 1]$.



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Create and Try the Test ...

... without coding knowledge `run_Practice()`

Practice Assessment Introduction Upload your Test Adaptive Assessment Results

Welcome to the Adaptive Assessment Tools App

This application allows you to conduct adaptive assessments using the continuous Markov procedure by Falmagne and Delgion (CMP, 1988) from Knowledge Space Theory (KST) and/or Adaptive Testing System for Psychological Disorders (ATS-PD, Donatello et al., 2017) from Formal Psychological Assessment (FPA). Whether you are assessing knowledge, evaluating clinical conditions, or any other application, this app provides a structured, efficient, and accurate way to collect your data.

What is Adaptive Testing in KST and FPA?

Adaptive testing is a dynamic process that tailors the assessment to each individual based on their responses. In both Knowledge Space Theory and Formal Psychological Assessment, this means covering the state that represents the learner's or patient's abilities, among the collection of possible states on the structure.

The Underlying Structure

Both KST and FPA rely on an underlying structure—be it a knowledge structure in KST or a clinical structure in FPA. This structure guides the adaptive process, ensuring that each question or evaluation step is meaningful and relevant. The app can also accommodate a random presentation of questions, providing flexibility in how the assessment is conducted.

How the App Works

- 1. Upload Your Assessment:** On the second page, you can upload your assessment data. This includes questions and a skill map or clinical context, which should be in a CSV file format. Alternatively, if you have pre-created an assessment in R, you can upload an RDA file containing a variable of the 'assessment' type.
- 2. Adaptive Assessment:** The third page dynamically displays questions based on the adaptive algorithm. You can visualize the progress through a graph diagram, which highlights the likelihood of different states with color coding. This diagram helps to understand the current state. At the top of the diagram are the states that contain most of the items (with the whole collection of items on top), and at the bottom of the diagram are the states with fewer items (with the empty set on the bottom).
- 3. Results:** The final page provides a detailed outcome of the assessment. You'll see the cardinality of the recovered state (the score), the identified state (knowledge or clinical), and insights on potential changes in status. This comprehensive result allows for informed decision-making or further evaluation. Moreover, the result of the test can be downloaded and analyzed using R.

This app is designed to be a powerful tool for educators, psychologists, and clinicians, providing deep insights through adaptive assessments tailored to each individual's responses. Explore the pages and see how adaptive testing can enhance your understanding of knowledge or clinical states!

References

- Donatello, I., Spoto, A., Sembo, F., Bedaloni, S., Grandol, U., & Vidotto, G. (2017). ATS-PD: An Adaptive Testing System for Psychological Disorders. *Educational and Psychological Measurement*, 77(5), 792-815. <https://doi.org/10.1177/0013164416652188>
- Falmagne, J.-C., & Delgion, J.P. (1988). A class of stochastic procedures for the assessment of knowledge. *British Journal of Mathematical and Statistical Psychology*, 41, 1-23. <https://doi.org/10.1111/j.2044.8317.1988.tb00204.x>

Create and Try the Test ...

... without coding knowledge `run_Practice()`

The screenshot displays the 'Practice Assessment' interface with the following elements:

- Navigation Bar:** Practice Assessment | Introduction | Upload your Test | Adaptive Assessment | Results
- Left Panel (Configuration):**
 - Which dataset to use?**
 - K4Data
 - Upload your test (CSV file)
 - Choose CSV file:** Browse... (No file selected)
 - Random:**
 - Repetition:**
 - Header:**
 - Separator:**
 - Comma
 - Semicolon
 - Tab
 - Quote:**
 - None
 - Double Quote
 - Single Quote
 - Delimiter:**
 - Comma
 - Period
- Download Test .RData:** Download

- Right Panel (Preview):**
- Text input: [X] "Data are not ok"
- Section: Your data
- Section: Test Parameters

Create and Try the Test ...

... without coding knowledge `run_Practice()`

Practice Assessment | Introduction | Upload your Test | Adaptive Assessment | Results

Which dataset to use?

RData

Upload own test (CSV file)

Choose CSV File

Browse...

Upload content

Knowledge

Repetition

Header

Separator

Comma

Tab

Quote

None

Double Quote

Single Quote

Decimal

Comma

Period

Download Test.RData

Download

[1] "Data are ok"

Your data

Item	Maggiore	Misure	Uguaglianza	Frazione	Decimale	addizione.1.dita	addizione.2.dite	ripeto	correct	incorrect	incorrect1	incorrect2
Inserisci maggiore, minore o uguale 7 ____ 12	0	1	0	0	0	0	0	0	+	+	+	NA
Inserisci maggiore, minore o uguale 190 ____ 164	1	0	0	0	0	0	0	0	+	+	+	NA
Inserisci maggiore, minore o uguale 4.5 ____ 4.30	0	0	1	0	1	0	0	0	+	+	+	NA
Inserisci maggiore, minore o uguale 0.06 ____ 0.60	0	1	0	0	1	0	0	0	+	+	+	NA
Inserisci maggiore, minore o uguale 1/5 ____ 0.2	0	0	1	1	1	0	0	0	+	+	+	NA
Inserisci maggiore, minore o uguale 1/2 ____ 0.2	1	0	0	1	1	0	0	0	+	+	+	NA
20 + 40 =	0	0	0	0	0	1	0	0	60	24	42	6
32 + 15 =	0	0	0	0	0	1	1	0	47	45	37	50
50 + 22 =	0	0	0	0	0	1	1	1	40	50	30	NA
9 + 1 =	0	0	0	0	0	1	0	1	10	9	9	NA

Test Parameters

Knowledge Structure

```

1 2 3 4 5 6 7 8 9 10
000000000 0 0 0 0 0 0 0 0 0
010000000 0 1 0 0 0 0 0 0 0
100000000 1 0 0 0 0 0 0 0 0
001000000 0 0 1 0 0 0 0 0 0
000001000 0 0 0 0 0 1 0 0 0
110000000 1 1 0 0 0 0 0 0 0
                    
```

Parameters

```

beta eta
1 0.15 0.15
2 0.15 0.15
3 0.15 0.15
4 0.15 0.15
5 0.15 0.15
6 0.15 0.15
7 0.15 0.15
8 0.15 0.15
9 0.15 0.15
10 0.15 0.15
                    
```

Termination criterion: likelihood_maximization
with values of 0.5

Items

```

question
1 Inserisci maggiore, minore o uguale 7 ____ 12
2 Inserisci maggiore, minore o uguale 190 ____ 164
3 Inserisci maggiore, minore o uguale 4.5 ____ 4.30
4 Inserisci maggiore, minore o uguale 0.06 ____ 0.60
5 Inserisci maggiore, minore o uguale 1/5 ____ 0.2
6 Inserisci maggiore, minore o uguale 1/2 ____ 0.2
7 20 + 40 =
                    
```

Create and Try the Test ...

... without coding knowledge `run_Practice()`

The screenshot displays a web interface for an adaptive assessment. At the top, there are navigation tabs: "Practice Assessment", "Introduction", "Upload your Test", "Adaptive Assessment" (which is active), and "Results".

On the left side, there is a teal sidebar with the following content:

- A welcome message: "Welcome on the adaptive assessment page, you will engage in an adaptive assessment where questions are presented based on your previous answers." Below it, a note states: "At any point, you may choose to restart the assessment by clicking the 'Restart Assessment' button. Please note that restarting the assessment will delete all previous responses, resetting the evaluation process entirely. This allows you to begin the assessment anew, free from any prior input or results." A "Restart Assessment" button is visible.
- A section titled "Visualizing Your Progress" with a paragraph: "A key feature of this page is the interactive graph that visualizes the structure underlying the assessment. This graph represents each possible state, with color coding to indicate the probability of each state being your current one based on the responses observed so far." Below this are two bullet points:
 - Lighter colors represent states with a lower probability, suggesting they are less likely to match your current knowledge or clinical status.
 - Darker colors represent states with a higher probability, indicating a closer match to your current knowledge or clinical status.
- A final paragraph: "As you continue to answer questions, the colors of these states will dynamically update, providing you with a real-time visualization of the system's evolving understanding of your current state."

The main content area on the right is titled "Adaptive Assessment" and contains a vertical list of radio buttons with values 60, 42, 24, and 6. A "Confirm" button is located below this list.

Below the "Adaptive Assessment" section is a "Dynamic plot" which shows a complex network graph with numerous nodes and connecting lines, representing the state space of the assessment.

Create and Try the Test ...

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The screenshot shows a web interface for an assessment. At the top, there is a navigation bar with tabs: "Practice Assessment", "Introduction", "Upload your Test", "Adaptive Assessment", and "Results" (which is currently selected). Below the navigation bar, the page is divided into two main sections. The left section is a teal sidebar containing a welcome message and a "Show Me the Output" button. The right section is the main content area, which is light gray and contains the following information:

- The score**
The assessment ended with 4 questions.
The score of the individual is 4 out of 10.
The score has been estimate on 4 observed correct responses and 2 incorrect ones.
- Your current status**
The individual current status have a probability of 0.0286 and it contains the following problems:
Inserisci maggiore, minore o uguale 7 ___ 12
Inserisci maggiore, minore o uguale 290 ___ 164
Inserisci maggiore, minore o uguale 0.06 ___ 0.60
Inserisci maggiore, minore o uguale 1/2 ___ 0.2
20 + 40 =
32 + 15 =
- What can change in your status**
The problems that are not in the current status:
Inserisci maggiore, minore o uguale 4.3 ___ 4.30
Inserisci maggiore, minore o uguale 1/5 ___ 0.2
18 + 22 =
9 + 1 =

At the bottom right of the page, there are several navigation icons: a back arrow, a forward arrow, a search icon, and a refresh icon.

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RAISE - Robotics and AI for Socioeconomic Empowerment

- In collaboration with some middle schools in Lombardia and Liguria
- A pilot test covering the middle school program in mathematics

Tuning a Test with mycaas

21 items with multiple choice and 595 states

Simulation parameters:

- two termination rules with six stopping criteria
 $SC = \{.5, .6, .7, .8, .9, 1\}$
- simulate ten response patterns for each $K \in \mathcal{K}$
- lucky guess $\eta_q = .2$ & careless error $\beta_q = .15$



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Performance Analysis

Accuracy and Efficiency Indexes

Given:

- true knowledge state K^w of a student w ;
- the probability distribution \mathcal{L}_m at the end of the assessment
- the recovered knowledge state \hat{K}_m^w

The following indexes were computed across simulated subjects:

- 1 The **average number of questions asked**.
- 2 The **average maximum probability** $\bar{\mathcal{L}}_m(\hat{K}_m)$.
- 3 The **average Hamming distance** $\bar{D}_m(K, \hat{K}_m)$ computed by

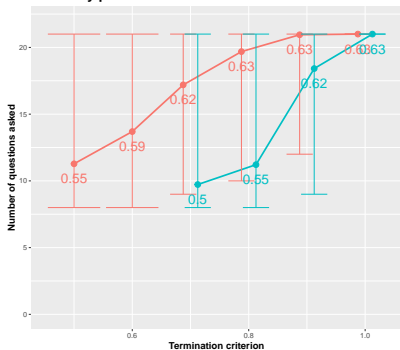
$$\bar{D}_m(K, \hat{K}_m) = \frac{1}{N} \sum_{w=1}^N |K^w \Delta \hat{K}_m^w|,$$

where Δ represents the symmetric set difference.

Performance Analysis

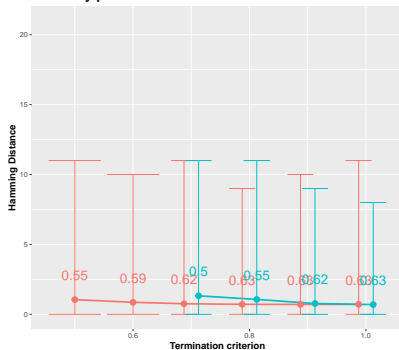
`performance_simulation(...)`

Efficiency plot



termination_rule likelihood_maximization items_discrimination

Accuracy plot



termination_rule likelihood_maximization items_discrimination

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`run_Assessment ()` **and** `run_Practice ()`
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 - **Response**: Performance analysis of the test
`performance_simulation ()`

Final Remarks

- The Shiny environment is used to make adaptive assessments more accessible and user-friendly.
- Currently the usability of the package is tested within RAISE in Liguria school
- <https://github.com/brancaccioandrea/mycass>



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