Rasch gone mixed: A mixed model approach to Implicit Association Test responses

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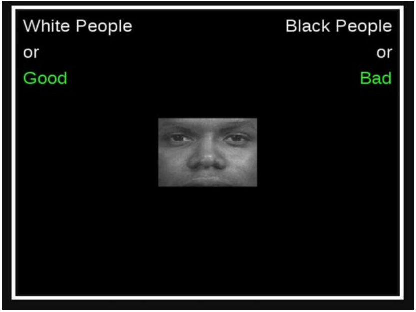
Contents

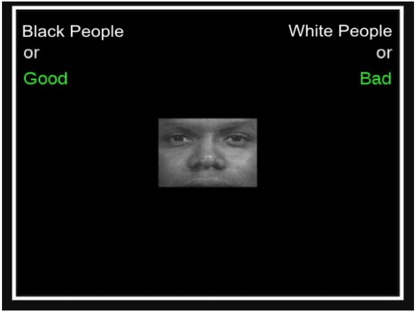
Introduction

What is the IAT?

IAT conditions



(a) Compatible.



(b) Incompatible.

Figure 1: IAT conditions.

Table 1: Race IAT Blocks

Block	Trials	Function	Left key	Right Key
1	20	Practice	White People (WP)	Black People (BP)
2	20	Practice	Good	Bad
3	60	Test	WP + Good	BP + Bad
4	20	Practice	BP	WP
5	60	Test	BP + Good	WP + Bad

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Compatible Condition

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Compatible Condition Incompatible Condition

The D score algorithm (Greenwald et al., 2003)

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$$D_{score} = \frac{M_{inc} - M_{comp}}{SDpooled_{comp,inc}}$$

The D score algorithm (Greenwald et al., 2003)

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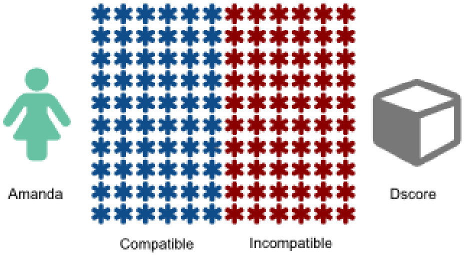
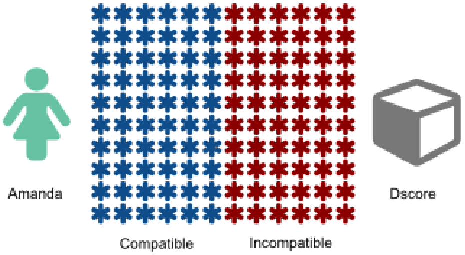
How do we read it?
Positive score: On average, slower responses in the incompatible condition than in the compatible condition
Negative score: On average, slower responses in the compatible condition than in the incompatible condition

The structure of the IAT


Amanda
Compatible
Incompatible
Dscore



Problems:

- Multiple observations on the same item by the same participant
- Different sources of variability
- Local dependence

Aims of the study

1. Generalized Linear Mixed Effects Models to IAT response accuracy → Rasch Model
2. Linear Mixed Effects Models to IAT (log) transformed time responses → Log Normal Model for Speed

Rasch and GLM

Rasch Model

$$logit(P(Y_{ij} = 1|(\theta_i, b_j)) = log\left(\frac{P(Y_{ij} = 1|(\theta_i, b_j)}{P(Y_{ij} = 0|(\theta_i, b_j)}\right)$$

$i = 1, \ldots, p$ Participants

$j = 1, \ldots, s$ Stimuli

Rasch Model

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$i = 1, \ldots, p$ Participants

$j = 1, \ldots, s$ Stimuli

Generalized Linear Model - GLM

$$x_i\beta_i = \eta_i = g(\mu_i)$$

x_i = i-th row of $p \times s$ matrix $\mathbf{X}$
μ_i = expected response
g = link function
η_i = linear predictors for the i-th response

Generalized Linear Model - GLM

For a binomial response:

$$x_i\beta_i = \eta_i = g(\mu_i) = log\left(\frac{\mu_i}{1-\mu_i}\right)$$

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Generalized Linear Mixed Effects Model - GLMM

$$\eta = X\beta + Za$$

β = Fixed effects
a = Random effects
Z = $p \times q$

Hierarchical Model for accuracy - response time

van der Linden (2007):

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First Level: Accuracy

Any IRT model
e.g.: Rasch model

$$P(Y_{ij} = 1|\theta_i, b_j) = \frac{exp(\theta_p - b_i)}{1 + exp(\theta_p - b_i)}$$

First Level: Response time

Log-normal model

$$ln(T_{ij}) = \beta_j - \tau_i$$

van der Linden (2007):

First Level: Accuracy

Any IRT model
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First Level: Response time

Log-normal model

$$ln(T_{ij}) = \beta_j - \tau_i$$

Second Level: Population Model

$$\xi_p = (\theta, \tau) \sim N\left[\begin{pmatrix}\mu_\theta \\ \mu_\tau\end{pmatrix}, \quad \begin{pmatrix}\sigma^2_\theta & \sigma^2_{\theta\tau} \\ \sigma^2_{\theta\tau} & \sigma^2_\tau\end{pmatrix}\right]$$

Second Level: Item Domain Model

$$\psi_i = (b_i, \beta_i) \sim N\left[\begin{pmatrix}\mu_b \\ \mu_\beta\end{pmatrix}, \quad \begin{pmatrix}\sigma^2_b & \sigma^2_{b\beta} \\ \sigma^2_{b\beta} & \sigma^2_\beta\end{pmatrix}\right]$$

Log Normal Model for response time

$$ln(T_{ij}) = \beta_j - \tau_i$$

β_j: Stimulus time intensity, $j = 1, \ldots, s$
τ_i: Person speed, $i = 1, \ldots, p$

Methodology

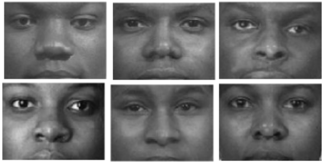
Race IAT

Valenced words (n=16)

- Positive words: Good, laughter, pleasure, glory, peace, happy, joy, love
- Negative words: Evil, bad, horrible, terrible, nasty, pain, failure, hate

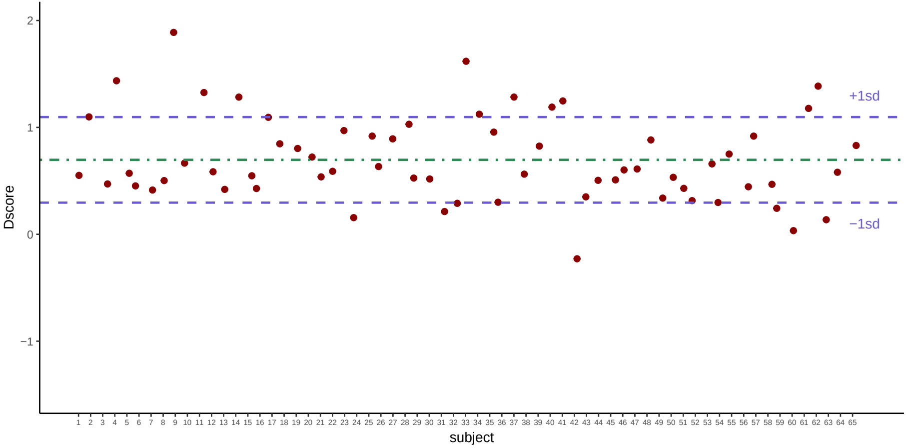
White Faces (M=3, F=3)

Black Faces (M=3, F=3)



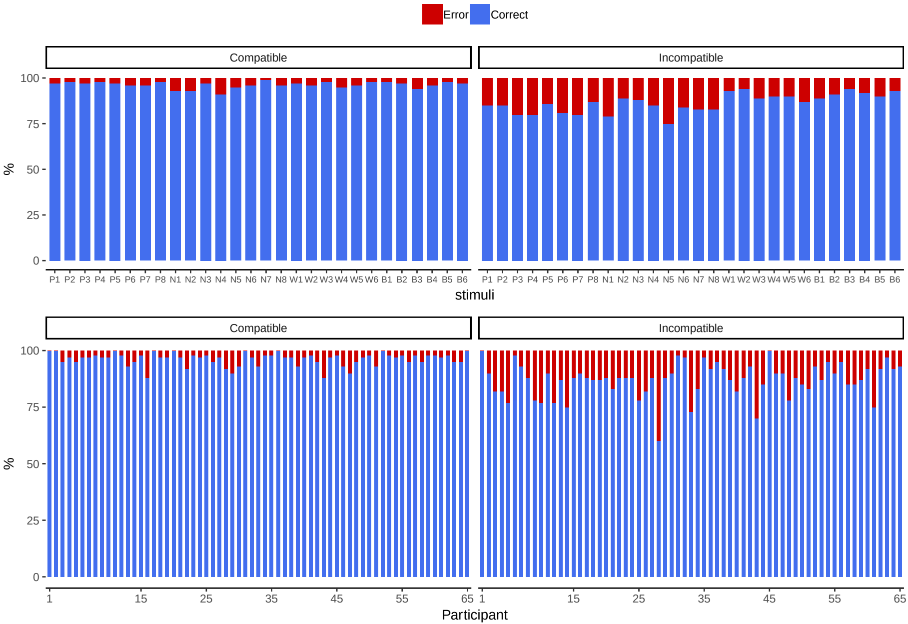
Participants

n = 65 (F = 49.23%, M_{age} = 24.95, SD_{age} = 2.09, $range_{age}$ = 19 - 30)



Accuracy

Take a look at the data - Accuracy


Error
Correct
Compatible
Incompatible
100
75
50
25
0
%
P1 P2 P3 P4 P5 P6 P7 P8 N1 N2 N3 N4 N5 N6 N7 N8 W1 W2 W3 W4 W5 W6 B1 B2 B3 B4 B5 B6
stimuli
Compatible
Incompatible
1 15 25 35 45 55 65
Participant

Model specification - Accuracy

Maximal Model

Random effects: Participants Intercept, Stimuli Intercept, Condition Random Slope in Participants, Condition Random Slope in Stimuli, Random Intercept interaction Participants*Stimuli

Fixed effects: Condition, $\beta_0 = 0$

$$\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + S_{0j} + P_{0i} * S_{0j} + (\beta_1 + P_{1i})c_k + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$$

$$i = 1, \ldots, P \text{ Participants}$$
$$j = 1, \ldots, S \text{ Stimuli}$$
$$k = 1, \ldots, C \text{ Conditions}$$

Model 1

Random effects: Condition Random Slope in Participants, Condition Random Slope in Stimuli

Fixed effects: Condition, $\beta_0 = 0$

$$\eta_{ij} = \beta_0 + \beta_{1k} + (\beta_1 + P_{1i})c_k + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$$

Model 2

Random effects: Participants (Intercept), Condition Random Slope in Stimuli, Interaction Stimuli * Participants

Fixed effects: Condition, $\beta_0 = 0$

$$\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + P_{0i} * S_{0j} + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$$

Model 3
Random effects: Participants (Intercept), Condition Random Slope in Stimuli
Fixed effects: Condition, $\beta_0 = 0$

$$\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$$

Model 4
Random effects: Participants (Intercept),Stimuli (Intercept)
Fixed effects: Condition, $\beta_0 = 0$

$$\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + S_{0j} + \epsilon_{ij}$$

Results - Accuracy

Model Comparison

Model	AIC	BIC	logLik	deviance	df.resid
M1	4142.05	4197.75	-2063.03	4126.05	7792
M2	4142.9	4191.63	-2064.45	4128.9	7793
M3	**4141.35**	**4183.12**	**-2064.67**	**4129.35**	**7794**
M4	4144.28	4172.12	-2068.14	4136.28	7796

Model 1: $\eta_{ij} = \beta_0 + \beta_{1k} + (\beta_1 + P_{1i})c_k + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$

Model 2: $\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + P_{0i} * S_{0j} + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$

Model 3: $\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$

Model 4: $\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + S_{0j} + \epsilon_{ij}$

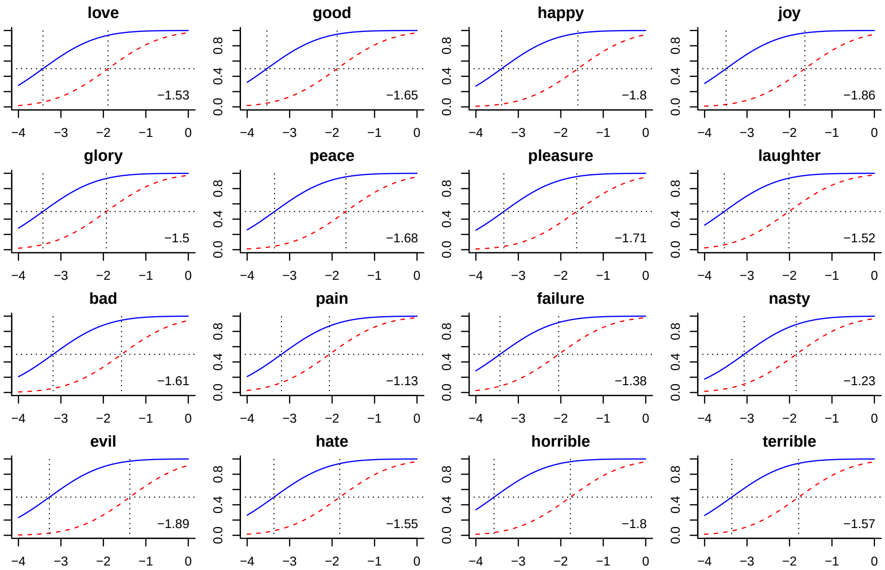
Model 3

$\eta_{ij} = \beta_0 + \beta_{1k} + P_{0i} + (\beta_1 + S_{1j})c_k + \epsilon_{ij}$

```
                      Estimate Std. Error
ConditionCompatible   3.418152  0.1226003
ConditionIncompatible 2.012852  0.1080449
 Groups      Name                  Std.Dev. Corr
 Participant (Intercept)           0.51184
 stimuli     ConditionCompatible   0.25983
             ConditionIncompatible 0.37184  0.166
```

ICC Model 3 Accuracy - Positive and Negative words

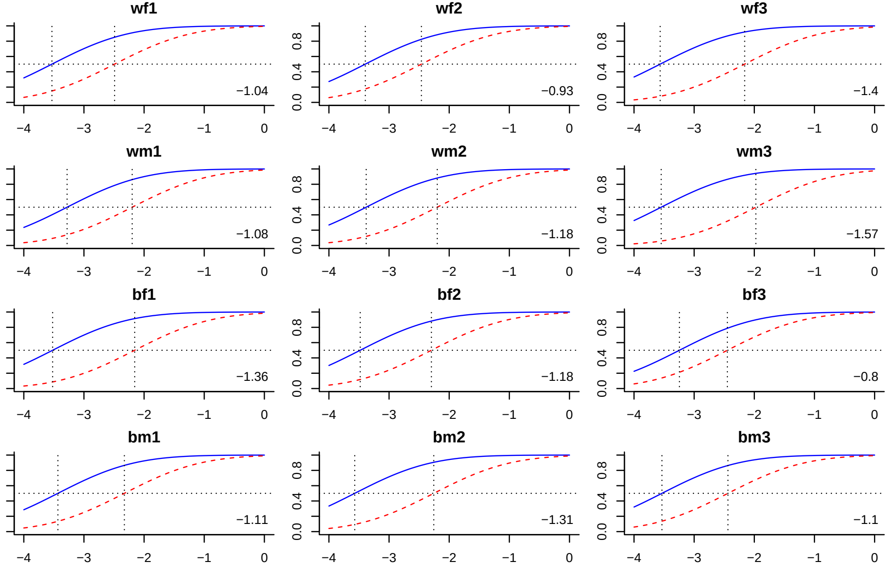
Compatibile Incompatible



ICC Model 3 Accuracy - Black and White Faces

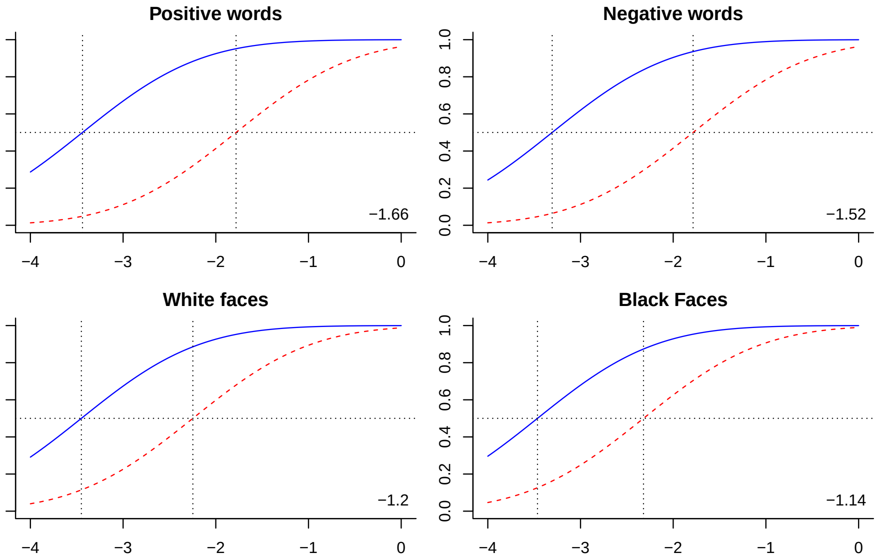
Compatibile Incompatible

"wf": White female, **"bf"**: Black female **"wm"**: White male, **"bm"**: Black male

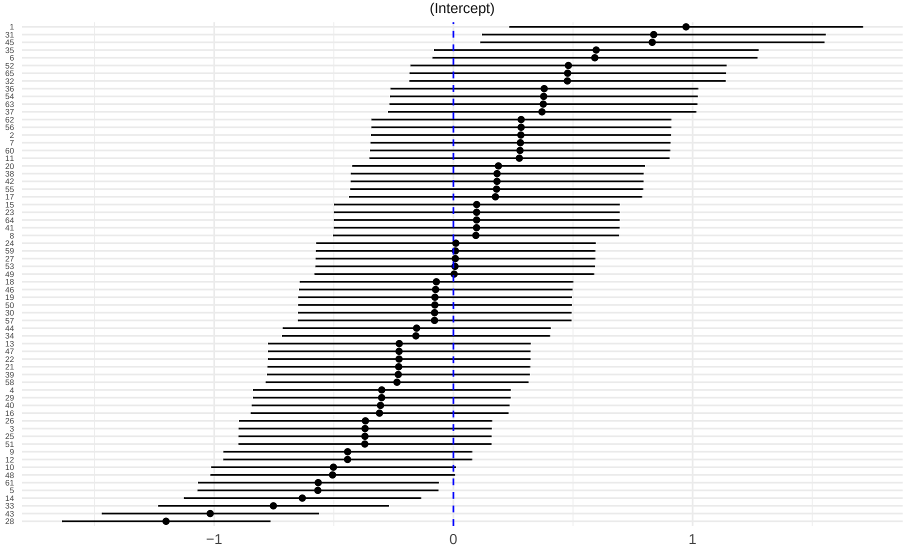


ICC Model 3 Accuracy - In a nutshell

Compatibile Incompatible

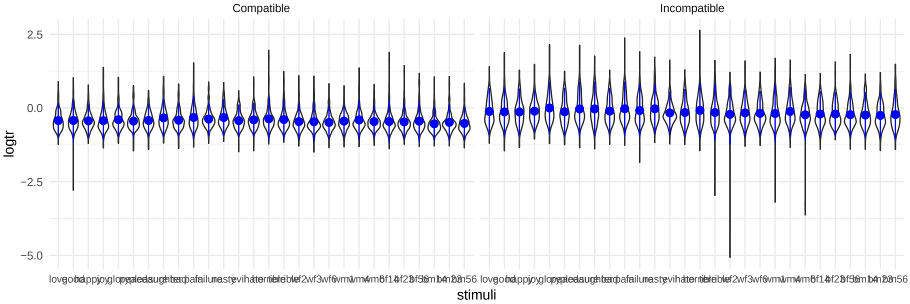


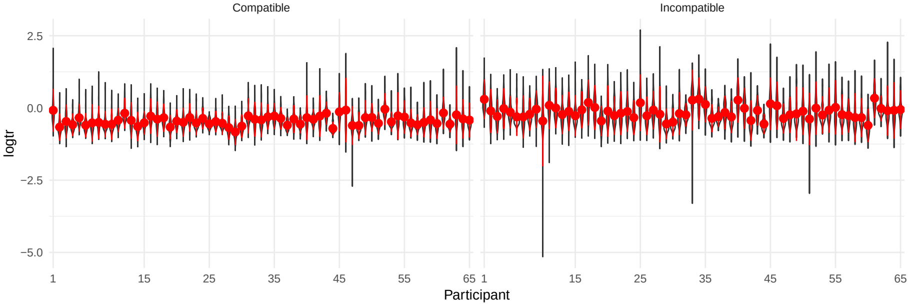
Model 3 Accuracy - Participants' Ability



Response Time

Take a look at the data - Response Time


Compatible
Incompatible
2.5
0.0
−2.5
−5.0
logtr
stimuli


Compatible
Incompatible
2.5
0.0
−2.5
−5.0
logtr
1
15
25
35
45
55
65
Participant

Model Specification - Response Time

Model 1

Random effects: Stimuli (Intercept), Condition Random Slope in Participants, Interaction Participants * Stimuli

Fixed effects: Condition, $\beta_0 = 0$

$$y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + S_{0j} * P_{0i} + (\beta_1 + P_{1i})c_k + \epsilon_{ij}$$

Model 2

Random effects: Stimuli (Intercept), Condition Random Slope in Participants

Fixed effects: Condition, $\beta_0 = 0$

$$y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + (\beta_1 + P_{1i})c_k + \epsilon_{ij}$$

Model 3

Random effects: Participants (Intercept), Stimuli (Intercept)

Fixed effects: Condition, $\beta_0 = 0$

$$y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + P_{0i} + \epsilon_{ij}$$

Results - Response Time

Model comparison - Response Time

Model	AIC	BIC	logLik	deviance	df.resid
M1	4703.85	4759.54	-2343.92	4687.85	7792
M2	**4705.62**	**4754.35**	**-2345.81**	**4691.62**	**7793**
M3	5113.45	5148.26	-2551.73	5103.45	7795

Model 1: $y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + P_{0i} * S_{0j} + (\beta_1 + P_{1i})c_k + \epsilon_{ij}$

Model 2: $y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + (\beta_1 + P_{1i})c_k + \epsilon_{ij}$

Model 3: $y_{ij} = \beta_0 + \beta_{1k} + S_{0j} + P_{0i} + \epsilon_{ij}$

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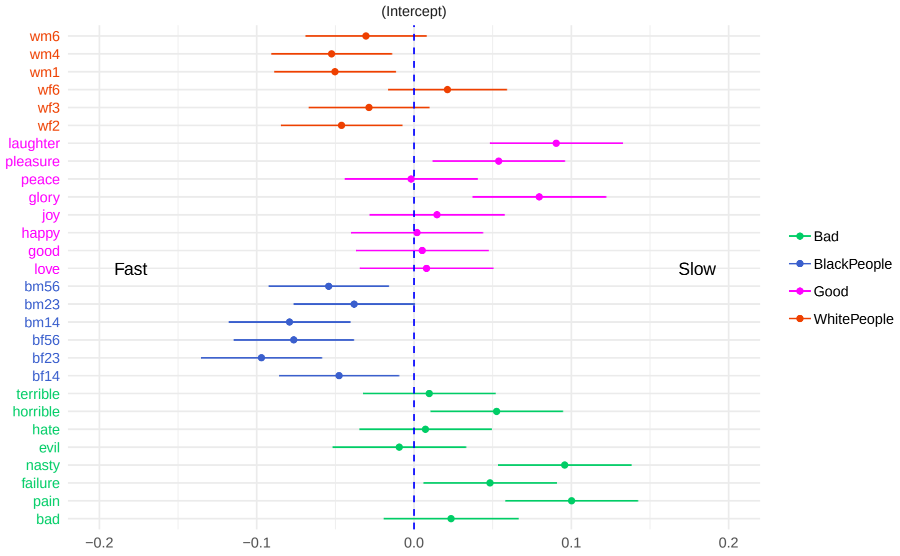
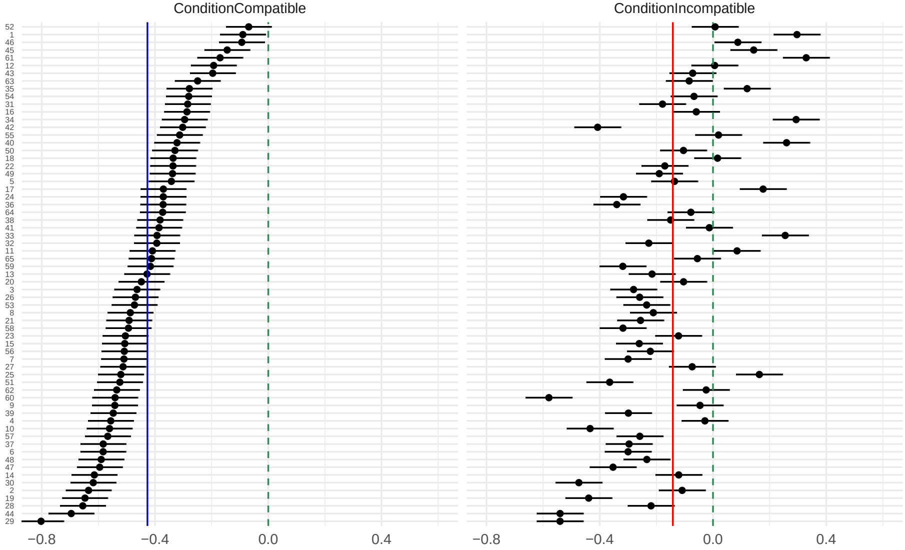
```
                        Estimate Std. Error
ConditionCompatible   -0.4267993 0.02311678
ConditionIncompatible -0.1418474 0.02915075
 Groups      Name                  Std.Dev. Corr
 Participant ConditionCompatible   0.15937
             ConditionIncompatible 0.21424  0.609
 stimuli     (Intercept)           0.05735
 Residual                          0.31791
```

Model 1 Response time - Stimuli Time Intensity



Model 1 Response time - Participants' Speed



Final remarks

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2. Future works:
   - Combine these parameters into one model (e.g., van der Linden)
   - Understand their relationship with relevant criteria (e.g., other IAT scoring methods or behavioural/explicit responses)
3. Replicate these models on other IAT data

Thank you!

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