Questionnaires and beyond: The Rasch model

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A_{Bart}

A_{Lisa}

A_{Bart}

Q1

$$4 + 5 = ?$$

d_{q1}

Q2

$$\frac{3}{2}x^2 + \frac{5}{4}x = ?$$

d_{q2}

A_{Lisa}

A_{Bart}

Q1

$$4 + 5 = ?$$

$$d_{q1}$$

Q2

$$\frac{3}{2}x^2 + \frac{5}{4}x = ?$$

$$d_{q2}$$

A_{Lisa}

$$\frac{A_p}{d_i} \tag{1}$$

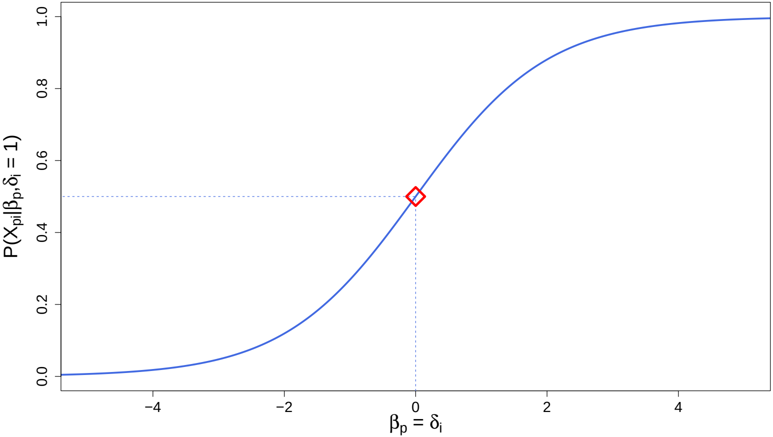
$$> 1 \text{ if } A_p > d_i$$

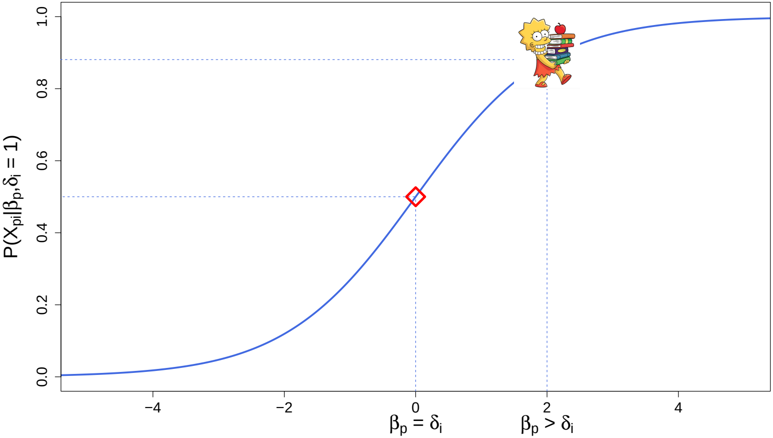
$$< 1 \text{ if } A_p < d_i$$

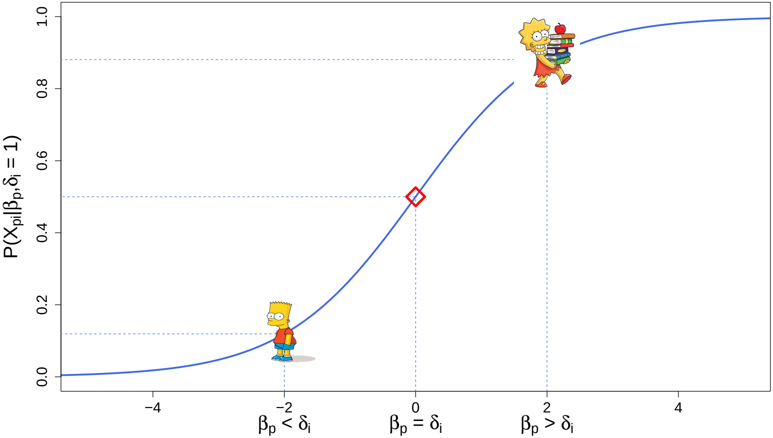
$$P(X_{pi} = 1) = \frac{\frac{A_p}{d_i}}{1 + \frac{A_p}{d_i}} \tag{2}$$

$$ln(A_p) = \beta_p \qquad ln(d_i) = \delta_i$$

$$P(X_{pi} = 1 | \beta_p, \delta_i) = \frac{exp(\beta_p - \delta_i)}{1 + exp(\beta_p - \delta_i)} \tag{3}$$


1.0
0.8
0.6
0.4
0.2
0.0
P(X_pi|β_p,δ_i = 1)
−4
−2
0
2
4
β_p = δ_i


$P(X_{pi}|\beta_p,\delta_i = 1)$
0.0
0.2
0.4
0.6
0.8
1.0
−4
−2
0
2
4
$\beta_p = \delta_i$
$\beta_p > \delta_i$

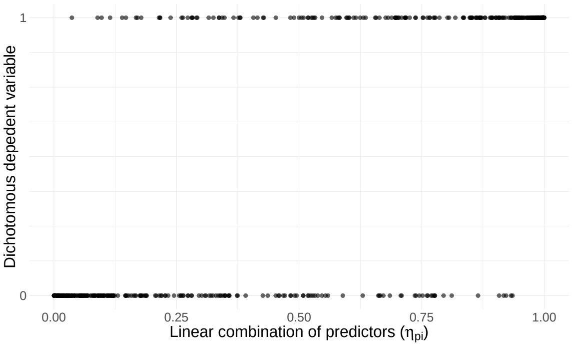


$P(X_{pi}|\beta_p,\delta_i = 1)$
0.0
0.2
0.4
0.6
0.8
1.0
−4
−2
0
2
4
$\beta_p < \delta_i$
$\beta_p = \delta_i$
$\beta_p > \delta_i$

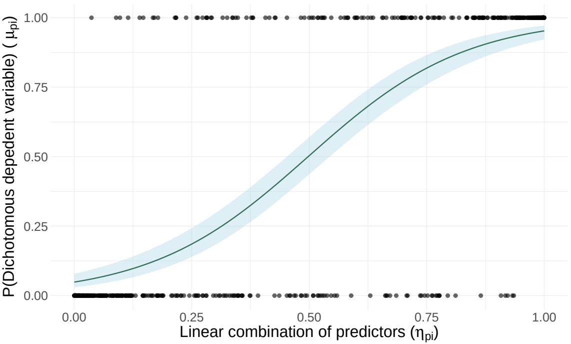
* Eureka moment *

$$P(X_{pi} = 1|\beta_p, \delta_i) = \frac{exp(\beta_p - \delta_i)}{1 + exp(\beta_p - \delta_i)}$$

Generalized Linear Model (GLM)
binomially distributed responses

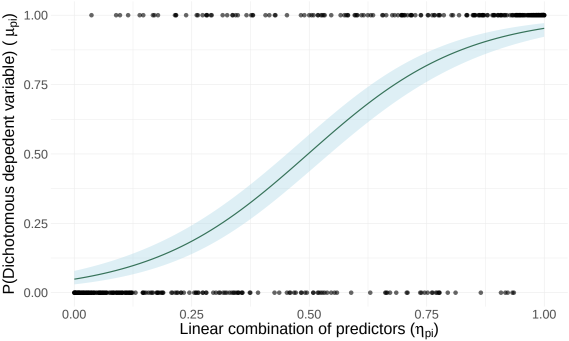


Generalized Linear Model (GLM)
binomially distributed responses



$$\mu_{pi} = g(\eta_{pi}) = log\left(\frac{\mu_{pi}}{1-\mu_{pi}}\right)$$

Generalized Linear Model (GLM)
binomially distributed responses



$$\mu_{pi} = g(\eta_{pi}) = log\left(\frac{\mu_{pi}}{1-\mu_{pi}}\right)$$

$$g^{-1} = \frac{exp(\eta_{pi})}{1+exp(\eta_{pi})}$$

Rasch model: Dichotomous responses

Issue

Quite limiting in Psychological Research

(Generalized) Linear Model: "Any" kind of response

e.g.: Response times

log-transformation and log-normal model parametrization

- **Linearity of the scores**

  Logarithm transformation → Respondents and items on the same latent trait

- **Comparison invariance**

  Respondents can be compared between each other without considering the items....and vice versa!

- **Local independence**

  Given the person → The responses to the items are independent

Unidimensionality

- **Linearity of the scores**

  Logarithm transformation → Respondents and items on the same latent trait

- **Comparison invariance**

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- **Local independence**

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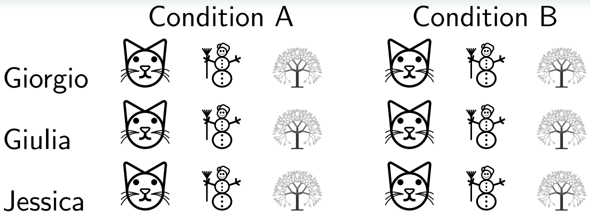
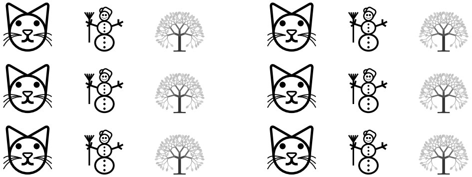

Condition A
Condition B
Giorgio
Giulia
Jessica

Condition A | Condition B

Giorgio
Giulia
Jessica



Local independence

Rasch model

- Can't be applied
- The estimates would make no sense

Generalized Linear Model

- Add the random part (Go Mixed)
- Obtain a Rasch-like parametrization of the data

Think outside of the box!

Yes	But
Rasch estimates	Rasch-like parametrization
The sky is the limit	Don't over complicate things
Keep it maximal	Keep it minimal

Think outside of the box!

Yes

Rasch estimates

The sky is the limit

Keep it maximal

But

Rasch-like parametrization

Don't over complicate things

Keep it minimal

Thank you

Questions!

https://ottaviae.github.io/AIP2022/Rasch/epifaniaRasch.pdf