Test length doesn't matter, it's how you use the items that counts: An intelligent procedure for item selection in Item Response Theory

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Sezione Sperimentale

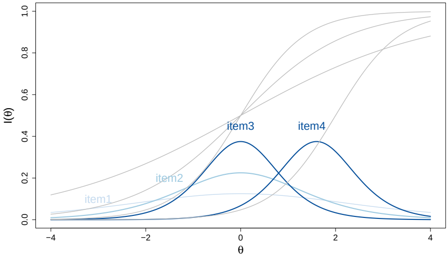
AIM

New automated procedure for item selection in IRT that only requires the definition of the desired characteristics of a test

Item and Test Information Functions

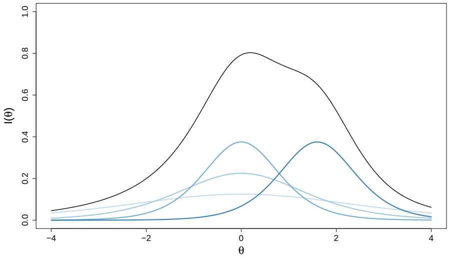
Item Information Function (IIF):

$$I_i(\theta) = a_i^2 P_i(\theta, b_i, a_i)[1 - P_i(\theta, b_i, a_i)]$$



Test Information Function (TIF):

$$I(\theta) = \sum_{i=1}^{N} I_i(\theta)$$

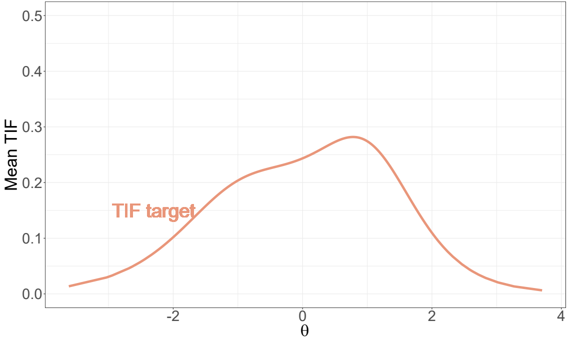


Item Locating Algorithm – ILA

B: Set of items of the item bank , items $i = \{1, 2, \ldots, ||B||\}$

TIF*: TIF target describing the desired characteristic of a test

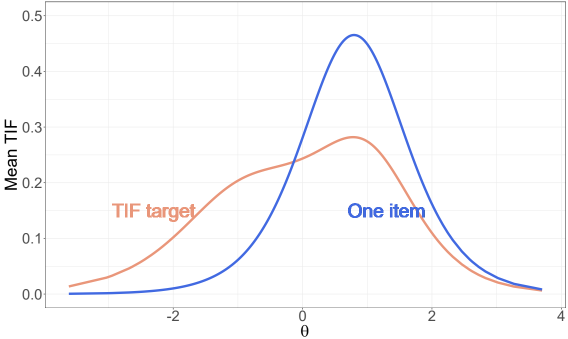
$Q^k_{\mathrm{ILA}} \subset B$: Set of items selected by ILA up to iteration k



$$k = 0,\ Q^0 = \emptyset,$$

$$\theta_{target} := \arg\max |\mathbf{TIF}^* - \mathbf{TIF}^0|, \text{ where } \mathbf{TIF^0} = (0, 0, \ldots, 0)$$

Item Locating Algorithm – ILA

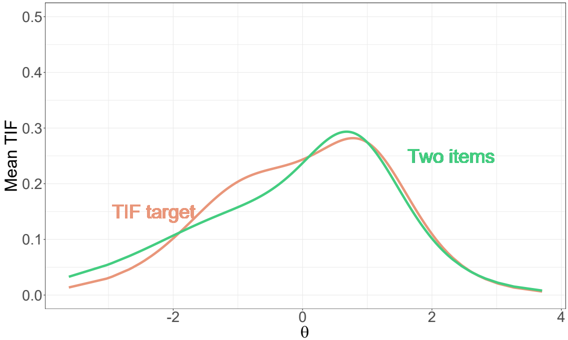


$$k = 1,\ Q^1 = Q^0 \cup \arg\min_{i \in B \setminus Q^0} |\theta_{target} - b_i|,\ ||Q^1|| = 1$$

$$|\mathbf{TIF}^* - \mathbf{TIF}^1| \geq |\mathbf{TIF}^* - \mathbf{TIF}^0| \rightarrow \texttt{false},\ k = 2$$

$$\theta_{target} := \arg\max |\mathbf{TIF}^* - \mathbf{TIF}^1|$$

Item Locating Algorithm – ILA

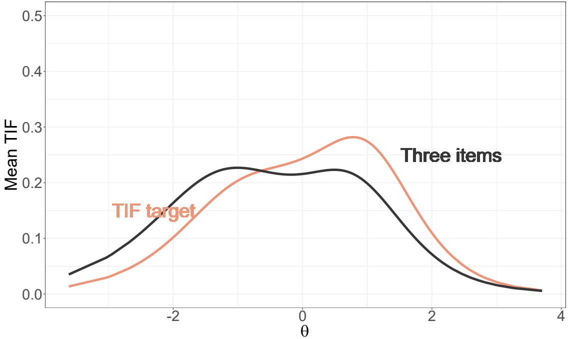


$$Q^2 = Q^1 \cup \arg\min_{i \in B \setminus Q^1} |\theta_{target} - b_i|, \; ||Q^2|| = 2$$

$$|\mathbf{TIF}^* - \mathbf{TIF}^2| \geq |\mathbf{TIF}^* - \mathbf{TIF}^1| \rightarrow \texttt{false}, \; k = 3$$

$$\theta_{target} := \arg\max |\mathbf{TIF}^* - \mathbf{TIF}^2|$$

Item Locating Algorithm – ILA



$$Q^3 = Q^2 \cup \arg\min_{i \in B \setminus Q^2} |\theta_{target} - b_i|, \ ||Q^3|| = 3$$

$$|\textbf{TIF}^* - \textbf{TIF}^3| \geq |\textbf{TIF}^* - \textbf{TIF}^2| \rightarrow \texttt{true} \rightarrow \textbf{end}, \ Q_{\textbf{ILA}} = Q^2$$

Brute Force Procedure – BFP

Item bank B

$Q_m \subset B$ item combinations of different lengths $l = 1, 2, \dots, N-1$

Total number of item combinations $2^{||B||} - 2$

$$\overline{\Delta}_{\mathbf{TIF}^{Q_m}} = mean(|\mathbf{TIF}^* - \frac{\sum_{i \in Q_m} IIF_i}{||Q_m||}|)$$

$$Q_{BFP} = \arg\min_{\emptyset \neq Q_m \subset Q} \overline{\Delta}_{\mathbf{TIF}^{Q_m}}$$

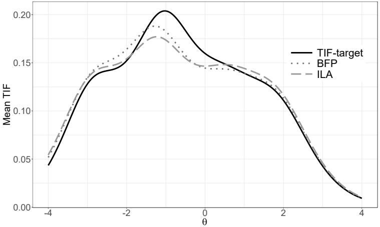
100 data frames:

1. Generate an item bank B of $N = 6$ items:
   - Difficulty parameters: $\mathcal{U}(-3, 3)$
   - Discrimination parameters: $\mathcal{U}(.90, 2.0)$
2. Random item selections of lengths l from B ($M_l = 3.34 \pm 1.13$) + modification parameters $\mathcal{U}(-0.20, 0.20) \rightarrow \mathbf{TIF}^*$
3. Considering $\mathbf{TIF}^*$ at Step 2 and item parameters at Step 1:
   - ILA $\rightarrow$ *Forwardly searches*
   - BFP $\rightarrow$ *Systematically tests*

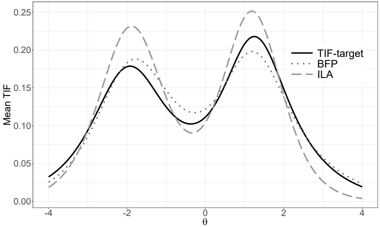
Comparison:

- $||Q_{\text{BFP}}|| - ||Q_{\text{ILA}}||$
- Percentile rank (RP) of the distance $\mathbf{TIF}_{\text{BFP}} - \mathbf{TIF}_{\text{ILA}}$

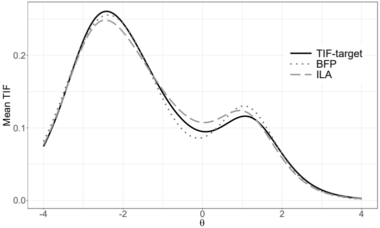
$$||Q_{\mathrm{BFP}}|| = ||Q_{\mathrm{ILA}}||,\ Q_{\mathrm{BFP}} \neq Q_{\mathrm{ILA}},\ RP = 3.17$$



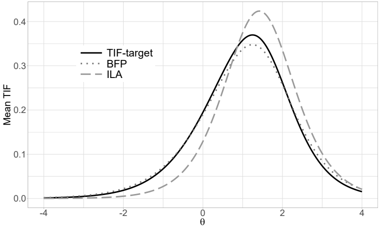
$$||Q_{\mathrm{BFP}}|| > ||Q_{\mathrm{ILA}}||,\ RP = 4.76$$



$$||Q_{\mathrm{BFP}}|| < ||Q_{\mathrm{ILA}}||,\ RP = 3.17$$



$$||Q_{\mathrm{BFP}}|| > ||Q_{\mathrm{ILA}}||,\ RP = 12.70$$



Pros of ILA

- It selects items that are able to recreate the desired characteristics of a test (usually)
- It is computationally "Light"

Cons of ILA

- It grounds its selection on a single θ_{target} at a time $\rightarrow$ it might select items minimizing the distance on that target but that are not very useful for the test
- It only forwardly searches an item $\rightarrow$ once it is in, it can't get out
- It does not account for the discrimination parameters of the items