Cut it short: A new item response theory-based approach for shortening tests

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Many items/questions in a questionnaire

Good

High assessment precision

High information/reliability

But

Respondent's fatigue

Response quality might be compromised

European Social Surveys

Cross-national survey carried on every two years since 2001

Assessment of attitudes, beliefs, and behavior patterns of diverse populations in different countries. Main focus $\rightarrow$ change/stability of:

- Living conditions
- Social structure
- Public opinion

Round 10:

Socio-demographic information

$+$

Well being, social exclusion, human values

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Item Response Theory

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2-PL Model

$$P(x_{pj} = 1|\theta_p, b_j, a_j) = \frac{exp[a_j(\theta_p - b_j)]}{1 + exp[a_j(\theta_p - b_j)]}$$

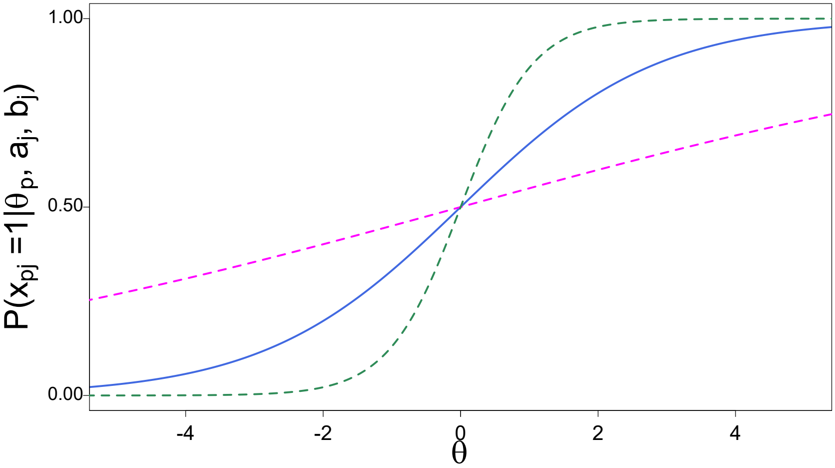
where:

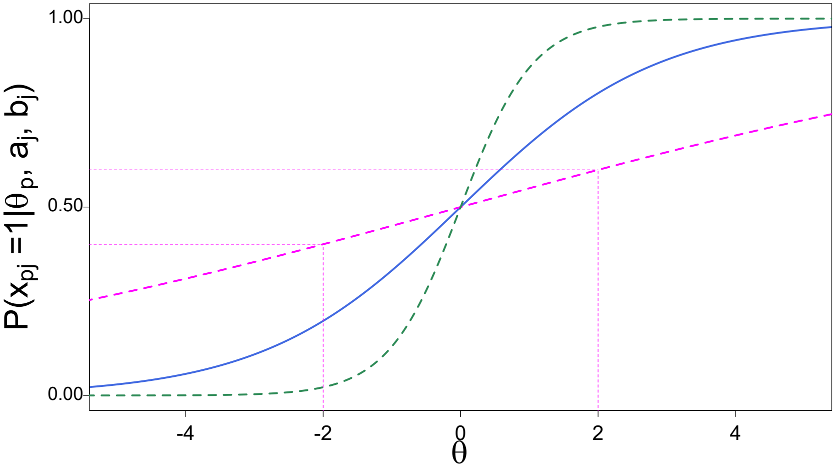
$P(x_{pj} = 1)$: Probability of endorsing item j by respondent p

θ_p: Ability of respondent p

b_j: Difficulty (location on the latent trait)of item j

a_j: Discrimination of item j


1.00
0.50
0.00
P(x_pj = 1|θ_p, a_j, b_j)
-4
-2
0
2
4
θ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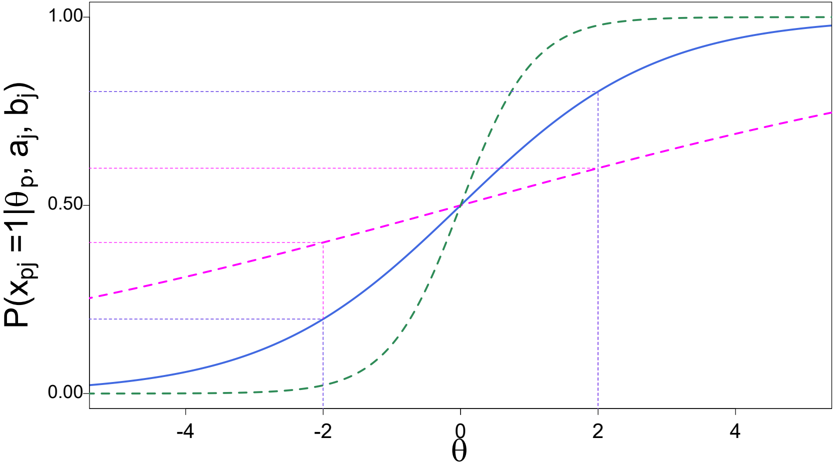
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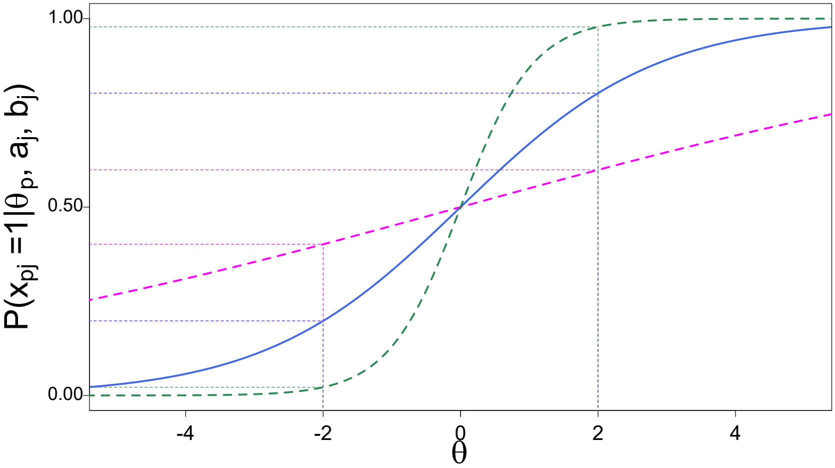
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Information functions

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$$IIF_j = a_j^2[P(\theta)(1 - P(\theta))]$$

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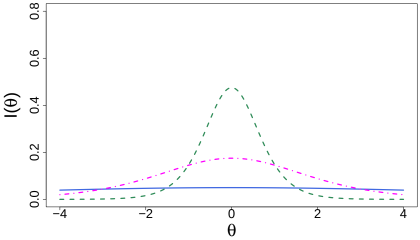

Figure 1: $a = 0.20$, $a = 0.70$, $a = 1.90$, $b = 0$

Information functions

Item Information Function

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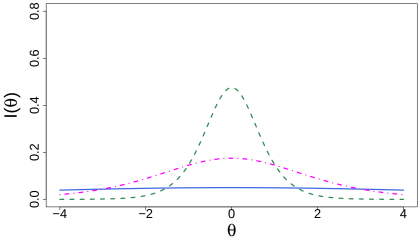
Test Information Function

$$TIF = \sum_{j=1}^{J} IIF_j$$



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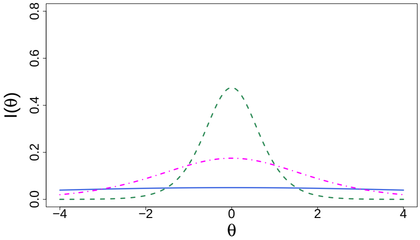

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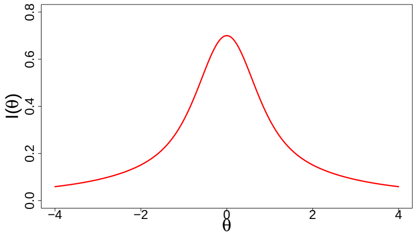

Figure 2: TIF $= IIF_1 + IIF_2 + IIF_3$

Benchmark procedure

Selected items → items with the highest *IIF*s

e.g.: 3-item short form from 10-item full-length test

item	b	a	IIF
1	−0.67	0.71	0.08
2	0.50	1.19	0.15
3	−2.43	0.25	0.01
4	2.12	1.98	0.24
5	1.72	0.39	0.03
6	−2.28	1.62	0.19
7	0.64	0.50	0.05
8	−2.51	1.68	0.19
9	−0.66	0.44	0.04
10	0.72	0.33	0.02

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θ-target procedures

Selected items $\rightarrow$ items with highest *IIF*s in respect to θ targets (θ')

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3	0.01	0.01	0.02
4	0.73	0.06	0.01
5	0.04	0.03	0.02
6	0.01	0.06	0.59
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Simulation

Create STFs (10-item, 30-item, 50-item, 70-item, 90-item) from a full-length test of 100 items:

100 items j:

- $b \sim \mathcal{U}(-3, 3)$
- $a \sim \mathcal{U}(0.40, 2)$

1000 respondents p

1. Normal distribution
   $p \sim \mathcal{N}(0, 1)$
2. Positive skewed distribution
   $p \sim Beta(1, 100)$ (linearly transformed to obtain negative values)
3. Uniform distribution
   $p \sim \mathcal{U}(-3, 3)$

An overall look

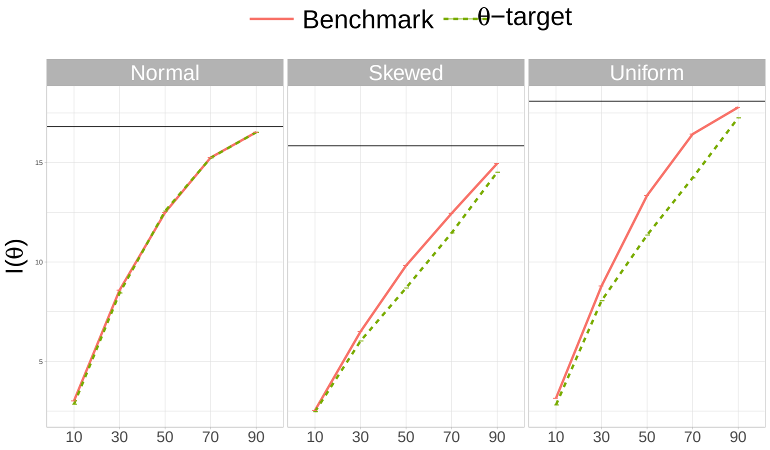

Figure 3: Overall Information of the short test forms

A closer look

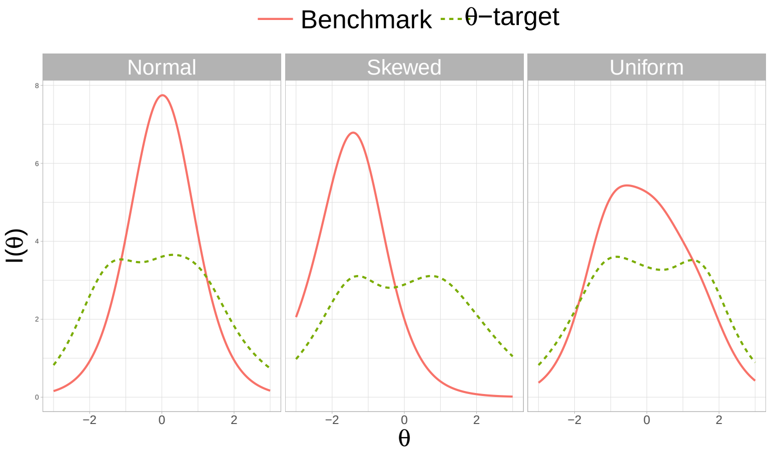

Figure 4: TIF of the 10-item short test form

- Item response theory provides a valid framework for shortening tests without losing information and reliability
- Targeting vs. ordering: There is no "one-fits-all" solution
- In the future $\rightarrow$ Which is the ideal number of item?

Thank you!

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