

L'importanza di essere significativo: Un esempio basato sul test della Torre di Londra

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La psicometria tra oggi e domani:
Sfide e nuovi orizzonti

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The ratio between the measures of a and b is constant and independent of the measurement unit:

$$\frac{\varphi(a)}{\varphi(b)} = \frac{\varphi'(a)}{\varphi'(b)},$$

where φ and φ' are two different scales of measurement of the same variable ¹.

¹Strictly referring to extensive physical measures

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Meaningful comparisons

The comparison between a and b is meaningful if it is invariant under all the unit transformations.

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Meaningful comparisons 2.0

Given that there is a difference between a and b , is this difference significant (or not) regardless of the scales of measurement?

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Admissible and non-admissible transformations

$$\varphi(P) = [0, 1, 2, 3]$$

$$\varphi'(P) = [0, 2, 4, 10]$$

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	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9
	φ								
Joe	0	1	2	2	2	3	3	3	3
Jane	0	2	2	2	3	3	3	3	3
Max	0	1	0	2	3	3	3	3	3
	φ'								
Joe	0	2	4	4	4	10	10	10	10
Jane	0	4	4	4	10	10	10	10	10
Max	0	2	0	4	10	10	10	10	10
	ε								
Joe	0	2	2	2	2	3	3	3	3
Jane	0	2	2	2	3	3	3	3	3
Max	0	2	0	2	3	3	3	3	3

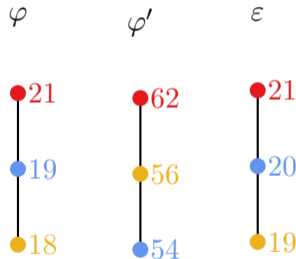
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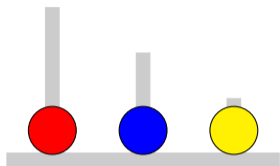
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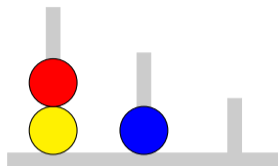
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The Tower of London Test (ToL Test)

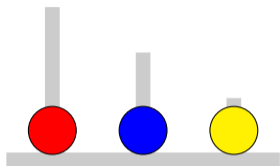


Starting configuration

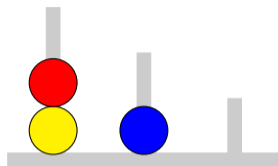


Goal configuration

The Tower of London Test (ToL Test)



Starting configuration



Goal configuration

Problem	Minimum moves	Alternative paths
Example	2	1
1	2	1
2	2	1
3	3	2
4	3	1
5	4	2
6	4	1
7	4	1
8	4	1
9	5	2
10	5	1
11	5	1
12	5	2

Attempt-based SMs

Scoring system	First attempt	Second attempt	Third attempt	Fourth on	Total sum score
KR	3	2	1	0	0 – 36
SH1	1	0			0 – 12

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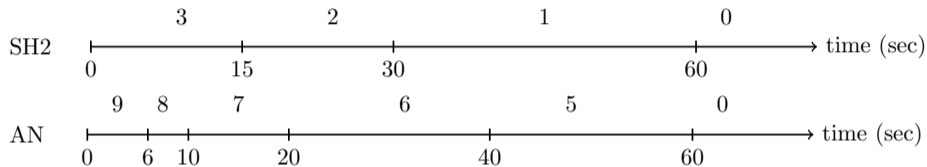
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P1	3	2		0	0 – 36
P1'	3	1		0	0 – 36

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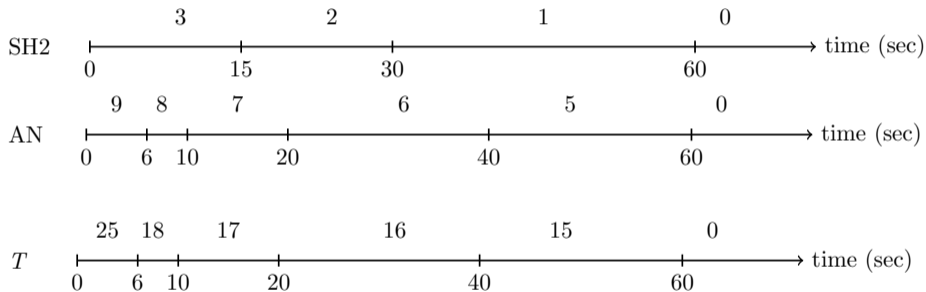
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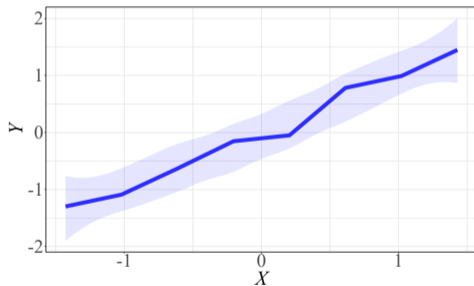
Latency-based SMs



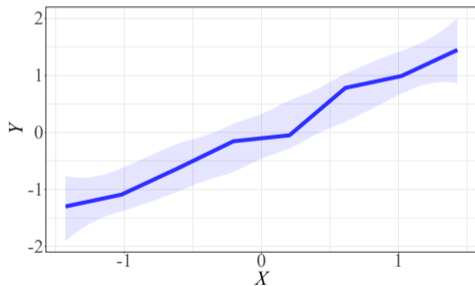
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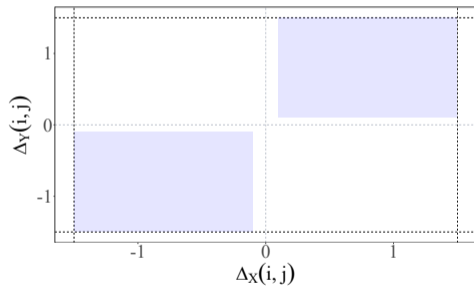
Monotonic relation



Monotonic relation

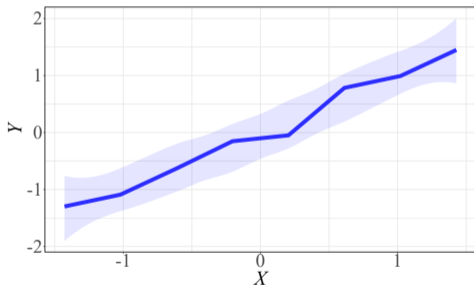


Distances and inversions

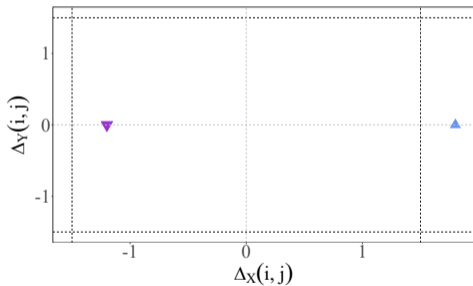


Methods: Individual differences

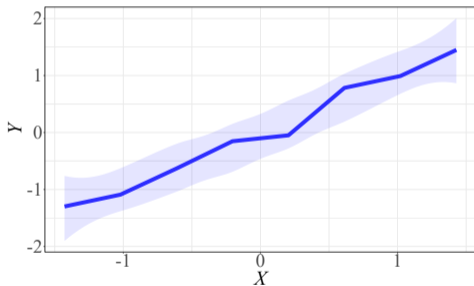
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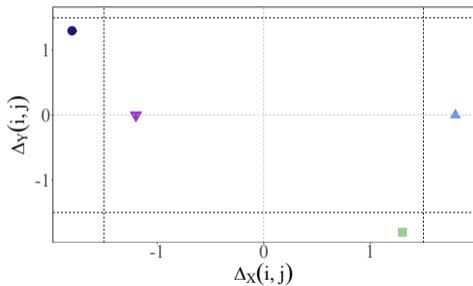
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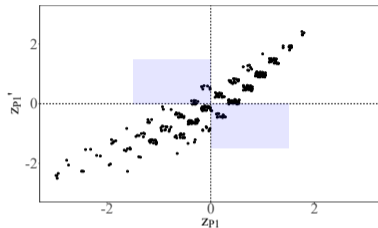
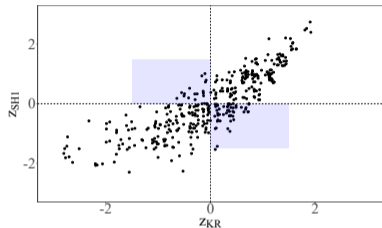


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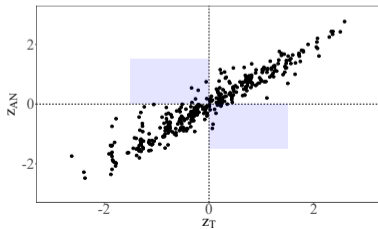
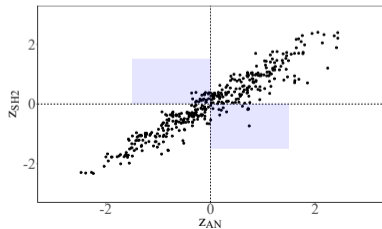


Results: Monotonic relation

Attempt-based SMs

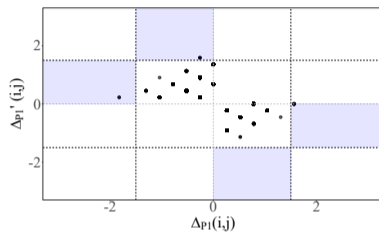
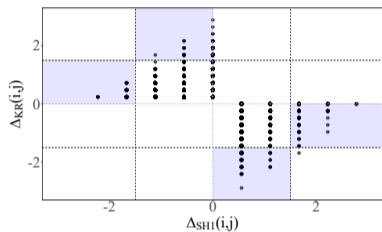


Latency-based SMs

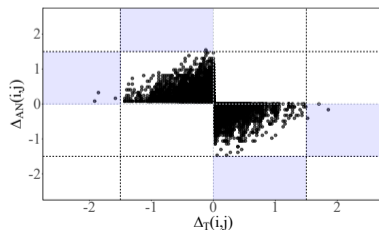
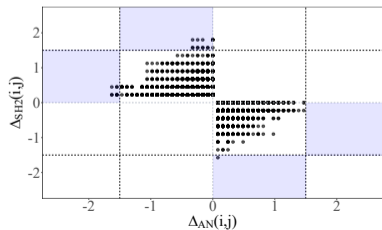


Results: Differences and distances

Attempt-based SMs



Latency-based SMs



$$H_0: \mu_{g1} - \mu_{g2} = 0$$

$$H_1: \mu_{g1} - \mu_{g2} \neq 0$$

t-test on the standardized scores considering different grouping variables:

Grouping variable	n_1	n_2
Gender	199	196
Administration order	202	193
Administration modality	211	184
Schooling years	171	224

Results: Attempt-based SM

	KR	SH1	P1	P1'
	<i>d</i>	<i>d</i>	<i>d</i>	<i>d</i>
Gender	1.84	2.11*	1.69	2.03*
	0.19	0.21	0.17	0.20
Test order	-0.15	0.80	-0.48	0.28
	-0.01	0.08	-0.05	0.03
Adm. Modality	-2.85**	-1.93	-2.69**	-2.35*
	-0.29	-0.19	-0.27	-0.24
Schooling	3.95***	3.56***	3.82***	3.85***
	0.39	0.36	0.38	0.39

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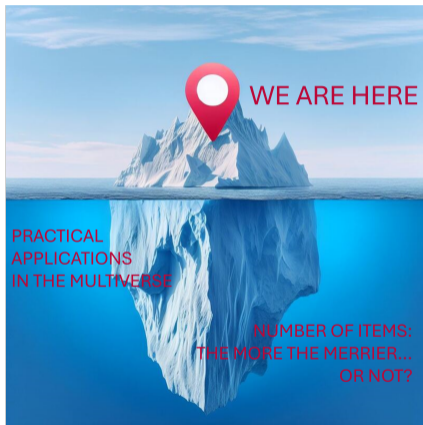
	SH2	AN	T
	<i>d</i>	<i>d</i>	<i>d</i>
Gender	1.64	1.88	2.10*
	0.17	0.19	0.21
Test order	0.37	0.99	0.95
	0.04	0.10	0.10
Adm. Order	-2.90**	-2.33*	-2.84**
	-0.29	-0.23	-0.29
Schooling	5.52***	5.32***	5.13***
	0.56	0.54	0.52

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Are we sure sum scores are a good idea...?


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RECOGNIZE THE VALUE OF THE SUM SCORE, PSYCHOMETRICS' GREATEST
ACCOMPLISHMENT

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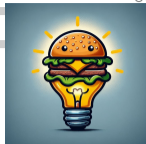
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Sum scores of ordinal data bring to a multiverse of contrasting results

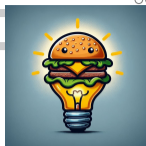


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Increasing the number of items does not solve the issue.... it worsens it!

Meaningfulness of psychological measures and reproducibility are interlaced

Research funded by the project “Computerized, Adaptive and Personalized Assessment of Executive Functions and Fluid Intelligence” (PRIN 2020, Prot. 20209WKCLL, P.I. Prof. Luca Stefanutti)



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Bright side:

Sum scores of truly dichotomous data (i.e., true vs. false, correct vs. incorrect) are meaningful

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